

# Welfare implications of strategic voting

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## References

- ▶ Aki Lehtinen. The welfare consequences of strategic behaviour under approval and plurality voting. *European J. Political Economy* 2008.

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- ▶ Lirong Xia and Vincent Conitzer. Stackelberg Voting Games. *Proceedings AAAI* 2010.
- ▶ Simina Brânzei, Ioannis Caragiannis, Jamie Morgenstern, Ariel Procaccia. How Bad is Selfish Voting? *Proceedings AAAI* 2013.

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- ▶ Voting games typically have enormously many Nash equilibria (for most rules, each unanimous profile is a NE).

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- ▶ Gibbard and Satterthwaite (1973–75) proved that if  $m \geq 3$  and  $n \geq 2$ ,  $R$  is onto and  $R$  is not a dictatorship, then for some preference profile,  $R$  is manipulable.

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- ▶ Much research has been carried out in order to understand how prevalent manipulability is, and how to minimize it.

## Aside: manipulation in the COMSOC literature

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- ▶ They tackled the slightly easier problem of **sensitivity** — how easy is it to change the winner?
- ▶ This work has been followed by a large number of papers investigating complexity of manipulation, mostly for special voting rules.
- ▶ Usually manipulation by coalitions is studied. Most common rules are easy to manipulate in this sense (tricky tie-breaking rules or weighted voters can make it hard).

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- ▶ Even if manipulability is something to be minimized, we know how to do that — dictatorship! Clearly there must be other things to (co)optimize, such as overall social welfare.
- ▶ Why not just study welfare?

## Simulation setup

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- ▶ We randomly generate a sample of sincere preference profiles (sometimes with explicit utilities), independently and uniformly. We use both random utilities in  $[0, 1]$  and utilities implied by the various rules.
- ▶ We use two measures of aggregate welfare: **egalitarian** (minimum) and **utilitarian** (mean). We also use **net satisfaction**: net fraction of voters who prefer the given strategic result to the sincere outcome. All measures are normalized.

## Behavioural assumptions

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- ▶ Simultaneous best-reply: naive Gibbard-Satterthwaite behaviour.
- ▶ 2-pragmatist: vote for your favourite among the top two in the sincere poll.
- ▶ Best-reply dynamics: repeatedly vote in fixed order until Nash equilibrium reached.

## Overview of results

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- ▶ Most profiles lead to sincere voting, since one player can't change the outcome.
- ▶ Many metrics are close to zero (the value for sincere voting).
- ▶ Overall welfare performance is best for SPNE, then IRM, then 2-pragmatism and naive best reply.
- ▶ Net satisfaction is usually positive for SPNE and IRM, but negative for the other solution concepts. However utilitarian welfare is (at least slightly) negative for all.

# Rank of the strategic winner

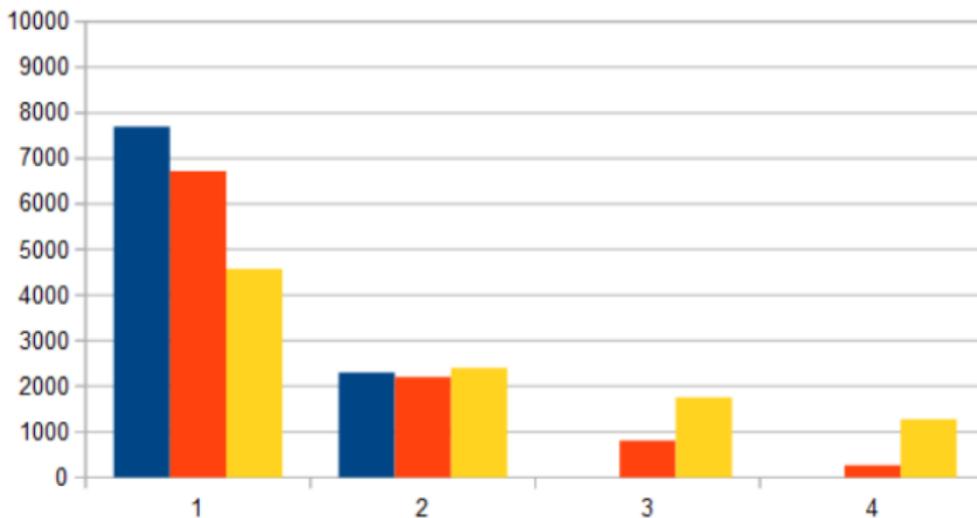


Figure : 2-pragmatist, plurality  $m = 4, n = 5$

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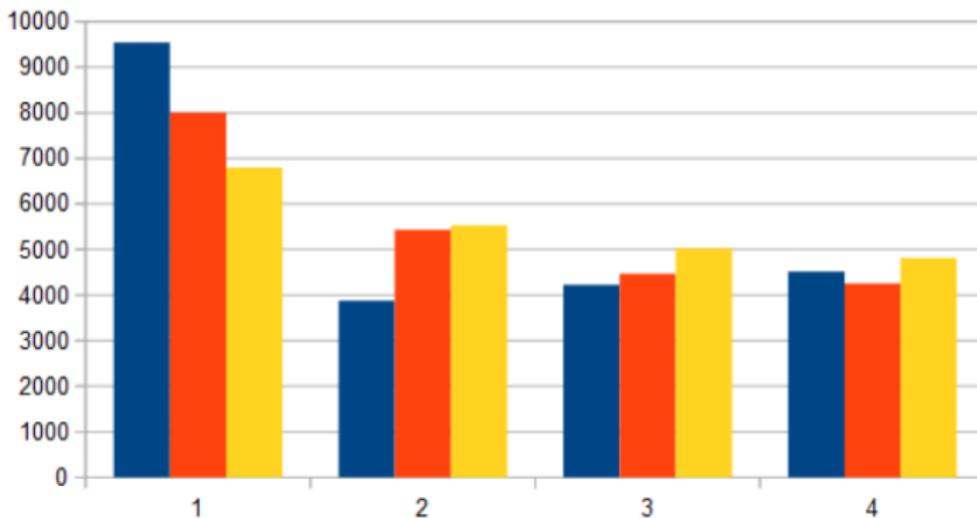
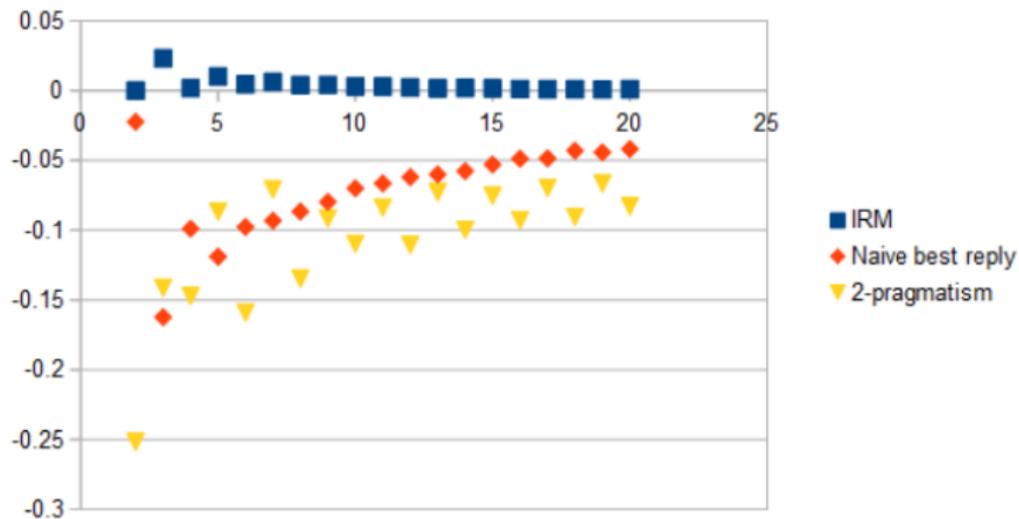


Figure : Naive best reply, plurality  $m = 4, n = 5$

## Net satisfaction

Figure : Borda,  $m = 4$ ,  $2 \leq n \leq 20$

## Previous work

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- ▶ Brânzei, Caragiannis, Morgenstern, Procaccia (2013) derived “price of anarchy” results for some rules under best-reply dynamics.

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- ▶ Strategic behaviour (by individual voters) is probably not an important issue, practically or theoretically. For large electorates, it seems less likely to occur. Even in small ones, it doesn't seriously reduce (and can even increase) overall welfare, unless voters are very naive in their beliefs about others.

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- ▶ The choice of a voting rule should perhaps be made on more classical criteria for an aggregation rule, related to the sincere model. And clever manipulation should perhaps be encouraged!

## Future work

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- ▶ Is it fruitful to consider evolutionarily stable strategies?
- ▶ What about strategic manipulation by coalitions (perhaps competing)?
- ▶ Do non-monotonic rules (such as instant runoff/alternative vote) give substantially different results?