

# Inefficiency in the Shadow of Unobservable Reservation Payoffs

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- ▶ Relax the first three.

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- ▶ In the Myerson and Satterthwaite (1983) world if there is no trade, the seller walks away with undisputed ownership of the object. Consequently the reservation payoff of the buyer is always 0 and that of the seller is his valuation.

## Example

Consider the problem of allocating some benefit to one of two players – player 1 and 2.

$$\theta_1 P(x_1, x_2) - x_1 \quad \text{and} \quad \theta_2(1 - P(x_1, x_2)) - x_2$$

where

$$P(x_1, x_2) = \frac{x_1^\lambda}{x_1^\lambda + x_2^\lambda} \quad \lambda \in (0, 1).$$

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Here the equilibrium payoff of each player would depend not only on her own type but also the type of the other player.

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- ▶ Such type dependence arises naturally when players have partial claims over the object and the claims are subject to enforcement through playing an inefficient default game such as a contest.

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- ▶ Literature on mechanism design with externalities where the payoff of players is not solely determined by whether or not they are allocated the good. Jehiel and Moldovanu (1999), Figueroa and Skreta (2009).
- ▶ Large literature on conflict. Skarperdas (2006) has a good survey.

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- ▶ Assumption:  $\theta_1 > \bar{\theta} > \underline{\theta} \geq 0$ .
- ▶ Since player 1 does not observe  $\theta$ , she treats the type of player 2 as a random variable  $\Theta$ .

## Reservation Payoffs

To participate, the expected payoffs must be weakly greater than the reservation payoffs which are

$$\mathbb{E}(v_1(\Theta)) \quad \text{and} \quad v_2(\theta)$$

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- ▶ We see that both players' reservation payoffs depend on the type of player 2. Since player 1 does not observe player 2's type, her reservation payoff is an expectation over  $\Theta$ .
- ▶ These payoffs may also depend on  $\theta_1$  but since this is publicly observed we can drop it to ease notation.

## Example

$$\theta_1 \mathbb{P}(\tilde{x}_1, \tilde{x}_2) - \tilde{x}_1 \quad \text{and} \quad \theta(1 - \mathbb{P}(\tilde{x}_1, \tilde{x}_2)) - \tilde{x}_2$$

where

$$\mathbb{P}(\tilde{x}_1, \tilde{x}_2) = \begin{cases} 1 & \text{if } \tilde{x}_1 = \tilde{x}_2 = 0 \\ \frac{\tilde{x}_1^\lambda}{\tilde{x}_1^\lambda + \tilde{x}_2^\lambda} & \text{otherwise, where } \lambda \in (0, 1) \end{cases}$$

- ▶ To simplify things assume that  $\theta$  takes only 2 values;  $\bar{\theta}$  with probability  $q$  and  $\underline{\theta} = 0$  with  $1 - q$ .

## Example

- ▶ We can solve out for the Bayesian-Nash equilibrium effort levels and payoffs.

$$x_1 := \operatorname{argmax}_{\tilde{x}_1} (\theta_1 \mathbb{E}(\mathbb{P}(\tilde{x}_1, x_2(\Theta)))) - \tilde{x}_1$$

$$x_2(\theta) := \operatorname{argmax}_{\tilde{x}_2} (\theta(1 - \mathbb{P}(x_1, \tilde{x}_2)) - \tilde{x}_2)$$

## Example

- ▶ Plugging back  $x_1$  and  $x_2(\theta)$  into the objective functions we can derive

$$\mathbb{E}(v_1(\Theta)) = \theta_1 \mathbb{E}(\mathbb{P}(x_1, x_2(\Theta))) - x_1$$

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$$v_2(\theta) = \theta(1 - \mathbb{P}(x_1, x_2(\theta))) - x_2(\theta)$$

- ▶ Although player 1 perceives her reservation payoff to be  $\mathbb{E}(v_1(\Theta))$ , her ex-post payoff when she faces a type  $\theta$  player 2 is

$$v_1(\theta) = \theta_1(\mathbb{P}(x_1, x_2(\theta))) - x_1$$

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$$\forall \theta \quad v_1(\theta) + v_2(\theta) < \theta_1 \quad (1)$$

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This along with  $\theta_1 > \bar{\theta}$  implies that ex-post efficiency requires that the object must always be allocated to player 1.

- ▶ However since player 2 is privately informed of his type it is possible that

$$\exists \theta \quad \mathbb{E}(v_1(\Theta)) + v_2(\theta) > \theta_1. \quad (2)$$

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- ▶ In the contest example (1) is always true because of the standard inefficiency of all contests.
- ▶ However (2) is satisfied as long as  $q$ , the probability with which player 2 is a high type, is low enough.
- ▶ This is because the unobservability of  $\theta$  constrains player 1's payoff to be  $\mathbb{E}(v_1(\Theta))$  without regard to the type of player 2. This will be the key to the inefficiency showcased in this model.

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Feasibility and budget balance imply

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$$\beta_1 + \mathbb{E}(\beta_2(\Theta)) \leq 1 \quad \text{and} \quad t_1 + \mathbb{E}(t_2(\Theta)) \leq 0$$

Since first best is attained only if the object is allocated to player 1,  $\beta_2(\theta) = 0$  must hold for all  $\theta$ .

# Result

## Proposition

*Full efficiency under budget balance is attainable if and only if  $\mathbb{E}(v_1(\Theta)) + v_2(\theta) \leq \theta_1$ , for all  $\theta \in [\bar{\theta}, \underline{\theta}]$ .*

# Result

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► **If –**

Set  $t = \max\{v_2(\theta)\}$ . This satisfies player 2's IR constraint. Since  $\mathbb{E}(v_1(\Theta)) \leq \theta_1 - \max\{v_2(\theta)\}$  the IR constraint of player 1 is also satisfied.

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► **Only If** –

Consider the case when there exists a  $\theta$  such that  $\mathbb{E}(v_1(\Theta)) + v_2(\theta) > \theta_1$ .  $t$  must be at least  $\max\{v_2(\theta)\}$  to satisfy the IR constraint of player 2. This however violates the IR constraint for player 1 since  $\mathbb{E}(v_1(\Theta)) > \theta_1 - \max\{v_2(\theta)\}$ .

## Second Best

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- ▶ To proceed further I assume that  $\theta$  is continuously distributed on the interval  $[\underline{\theta}, \bar{\theta}]$ .
- ▶ Recall that payoffs from the mechanism  $\mu_1$  and  $\mu_2(\theta)$  are composed of

$$\mu_1 = \beta_1 \theta_1 + t_1 \quad \text{and} \quad \mu_2(\theta) = \beta_2(\theta) \theta + t_2(\theta).$$

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$$\begin{array}{ll} \theta\beta'_2(\theta) + t'_2(\theta) = 0 & \Leftrightarrow \mu'_2(\theta) = \beta_2(\theta) \quad - \text{local incentive compatibility} \\ \beta'_2(\theta) \geq 0 & \Leftrightarrow \mu''_2(\theta) \geq 0 \quad - \text{monotonicity} \end{array}$$

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$$\begin{array}{lll} \theta\beta_2'(\theta) + t_2'(\theta) = 0 & \Leftrightarrow \mu_2'(\theta) = \beta_2(\theta) & \text{– local incentive compatibility} \\ \beta_2'(\theta) \geq 0 & \Leftrightarrow \mu_2''(\theta) \geq 0 & \text{– monotonicity} \\ 1 \geq \beta_2(\theta) \geq 0 & \Leftrightarrow 1 \geq \mu_2'(\theta) \geq 0 & \text{– feasibility} \end{array}$$

## Second Best – Optimization

- ▶ Solving for the surplus maximizing mechanism reduces to solving the following problem – for an exogenously given  $\mathbb{E}(v_1(\Theta))$  and a function  $v_2(\theta)$ ,

$$\min_{\beta_2(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} (\theta_1 - \theta) \beta_2(\theta) f(\theta) d\theta$$

subject to the constraints described above.

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subject to the constraints described above.

- ▶ If the reservation payoffs arise from a Bayesian game the solution to this problem is simple.

# Bayesian Reservation Payoffs

## Proposition

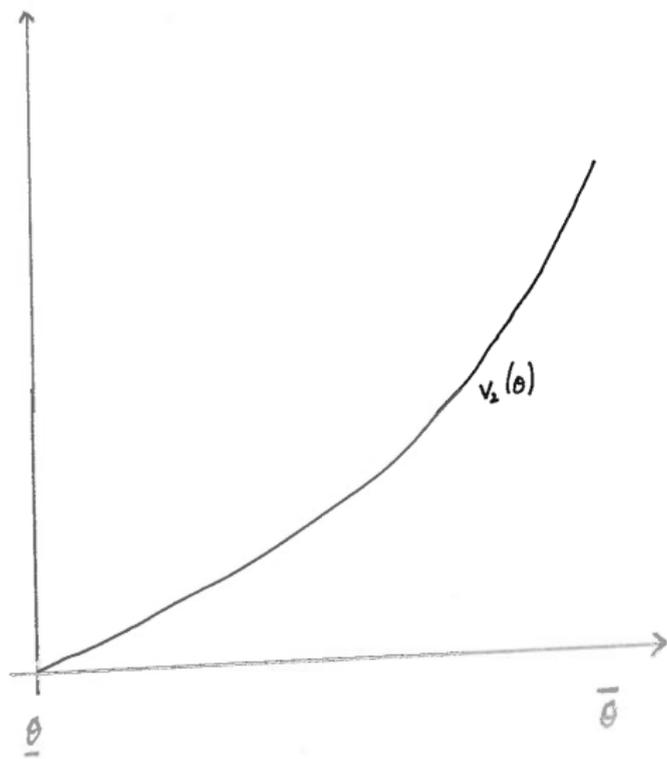
If  $\mathbb{E}(v_1(\Theta))$  and  $v_2(\theta)$  arise from a Bayesian game without third party subsidy, the surplus maximizing mechanism comprises of

$$\begin{aligned} \beta_2(\theta) &= v_2'(\theta) & \text{and} & & t_2(\theta) &= v_2(\theta) - \theta v_2'(\theta) & \text{if } \bar{\theta} \geq \theta > \hat{\theta} \\ \beta_2(\theta) &= 0 & \text{and} & & t_2(\theta) &= v_2(\hat{\theta}) & \text{if } \hat{\theta} \geq \theta \geq \underline{\theta} \\ \beta_1 &= 1 - \mathbb{E}(\beta_2(\Theta)) & \text{and} & & t_1 &= -\mathbb{E}(t_2(\Theta)) \end{aligned}$$

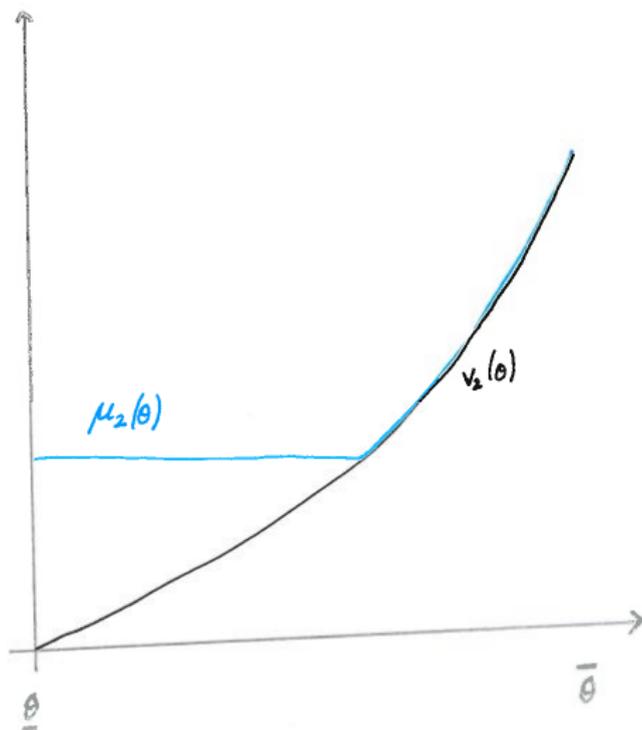
where  $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$  is the highest  $\theta$  that satisfies

$$\mathbb{E}(v_1(\Theta)) + v_2(\bar{\theta}) - \theta_1 = \int_{\hat{\theta}}^{\bar{\theta}} v_2'(\theta) \left( F(\theta) - (\theta_1 - \theta)f(\theta) \right) d\theta$$

# Bayesian Reservation Payoffs



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# General Reservation Payoffs

- ▶ When reservation payoffs arise an equilibrium of a Bayesian game, they satisfy all the constraints that we require  $\mu_2(\theta)$  to satisfy. In particular  $v_2(\theta)$  is guaranteed to be non decreasing, weakly convex, and  $1 \geq v_2'(\theta) \geq 0$ .

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- ▶ Assume that  $v_2(\theta)$  is twice differentiable and either concave or convex – this would subsume Bayesian reservation payoffs.

# General Reservation Payoffs

- ▶ To make progress we need to define the notion of a proximate implementable mechanism.

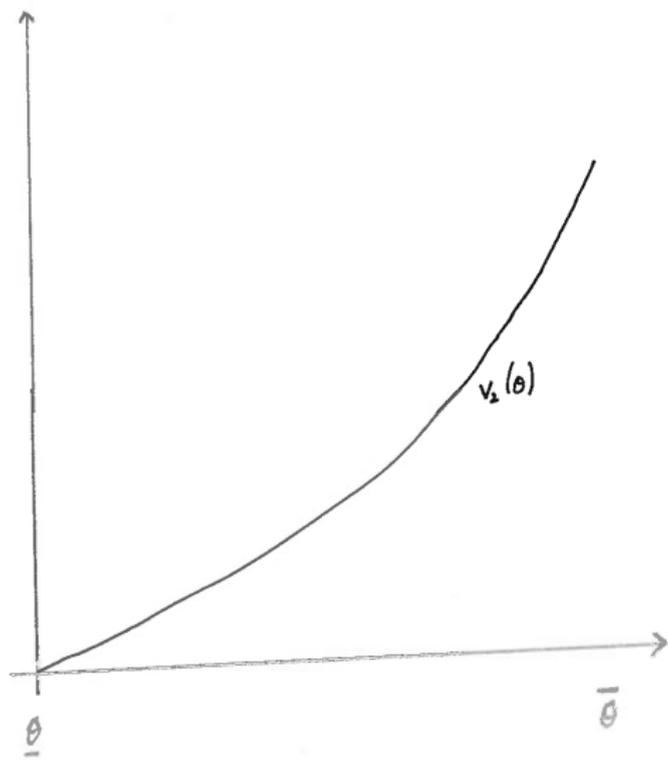
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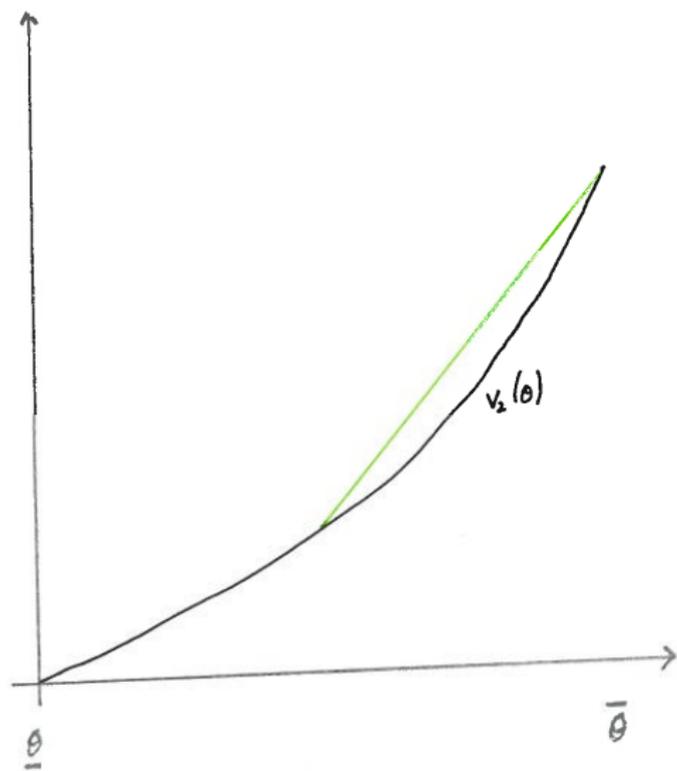
## Definition

A proximate implementable mechanism  $\eta(\theta)$  for  $\theta \in [\underline{\theta}, \bar{\theta}]$  is continuous, weakly convex, non-decreasing, and differentiable except at finitely many points, with  $1 \geq \eta'(\bar{\theta})$ , satisfies  $\eta(\theta) \geq v_2(\theta)$  and there does not exist another mechanism  $\tilde{\eta}(\theta)$  satisfying the same constraints such that  $\eta(\theta) \geq \tilde{\eta}(\theta)$  for all  $\theta$  and  $\eta(\theta) > \tilde{\eta}(\theta)$  for some  $\theta$ .

$$v_2'(\bar{\theta}) > 1, v_2''(\theta) > 0$$



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# General Reservation Payoffs

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# General Reservation Payoffs

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## Proposition

*Let  $\Psi' \subseteq \Psi$  be the subset of proximate implementable mechanisms that satisfy*

$$\mathbb{E}(\eta(\Theta)) - \int_{\underline{\theta}}^{\bar{\theta}} (\theta_1 - \theta)\eta'(\theta)f(\theta)d\theta \leq \theta_1 - \mathbb{E}(v_1(\Theta)). \quad (3)$$

*A surplus maximizing mechanism exists if and only if  $\Psi'$  is non empty.*

# Characterizing the Surplus Maximizing Mechanism

## Proposition

Assume  $v_2(\theta)$  is twice differentiable and either concave or convex,  $(\theta_1 - \theta)f(\theta)$  is non-increasing in  $\theta$ , and there exists an  $\eta(\theta)$  satisfying (3). The surplus maximizing mechanism must take the form

$$\begin{aligned} \beta_2(\theta) &= \mu_2'(\theta) & \text{and} & & t_2(\theta) &= \mu_2(\theta) - \theta\mu_2'(\theta) & \text{if } \bar{\theta} \geq \theta > \hat{\theta}, \\ \beta_2(\theta) &= 0 & \text{and} & & t_2(\theta) &= \mu_2(\hat{\theta}) & \text{if } \hat{\theta} \geq \theta \geq \underline{\theta}, \\ \beta_1 &= 1 - \mathbb{E}(\beta_2(\Theta)) & \text{and} & & t_1 &= -\mathbb{E}(t_2(\Theta)). \end{aligned} \quad (4)$$

where

$$\mu_2(\theta) = \operatorname{argmax}_{\eta(\theta) \in \Psi} \left( \int_{\hat{\theta}}^{\bar{\theta}} (\theta - \theta_1) \eta'(\theta) f(\theta) d\theta \right) \quad (5)$$

and  $\hat{\theta}$  is the highest value of  $\theta$  that satisfies

$$\mathbb{E}(v_1(\Theta)) = \theta_1 - F(\hat{\theta})\mu_2(\hat{\theta}) - \int_{\hat{\theta}}^{\bar{\theta}} (\theta_1 - \theta)\mu_2'(\theta)f(\theta)d\theta. \quad (6)$$

# Features of the Surplus Maximizing Mechanism

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- ▶ The uninformed player is always pushed down to her reservation payoff.
- ▶ The total surplus is decreasing in  $\mathbb{E}(v_1(\Theta))$  – the more lower the reservation payoff (the more inefficient the default game), the closer we get to the first best.

# Conclusion

- ▶ A simple condition turns out to be necessary and sufficient to guarantee that the first best is not attainable.

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- ▶ If the reservation payoffs arise from a Bayesian game, the second best is always implementable under budget balance – It is always possible to avoid an inefficient default game (but still not reach the first best)

# Conclusion

- ▶ A simple condition turns out to be necessary and sufficient to guarantee that the first best is not attainable.
- ▶ If the reservation payoffs arise from a Bayesian game, the second best is always implementable under budget balance – It is always possible to avoid an inefficient default game (but still not reach the first best)
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- ▶ The inefficiency of the second best is increasing in the reservation payoffs.