Fair division with probabilistic demand

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Motivation

- The classic bankruptcy/rationing problem
- Demand uncertainty
- Examples: budget allocation; network design
- This paper: probabilistic demands

The model

- I: a finite set of agents
- p_i, $i \in I$: a probability measure on $\mathbb{R}_+$ s.t. $\operatorname{supp}(p_i)$ is compact and $\max \operatorname{supp}(p_i) > 0$
- $T \in [0, \sum_{i \in I} \max \operatorname{supp}(p_i)]$: money to be divided
- A division problem: $(I, (p_i)_{i \in I}, T)$
- A solution: $t \in \mathbb{R}_+^I$ s.t. $\sum_{i \in I} t_i = T$ and $t_i \leq \max \operatorname{supp}(p_i)$ for all $i \in I$
- A division rule: $F(I, (p_i)_{i \in I}, T) = t$

An example

- $I = \{1, 2\}$
- $p_1(50) = 1$
- $p_2(100) = \frac{9}{10}$, $p_2(0) = \frac{1}{10}$
- $T = 50$
- A key feature is the possibility of losing resources when the realized demand is less than the allocated amount.
- The question is how to accommodate both fairness and non-wastefulness?

Axioms

- Anonymity: Suppose that $F(I, (p_i)_{i \in I}, T) = t$. If $p_i = p_j$ for $i, j \in I$, then $t_i = t_j$.

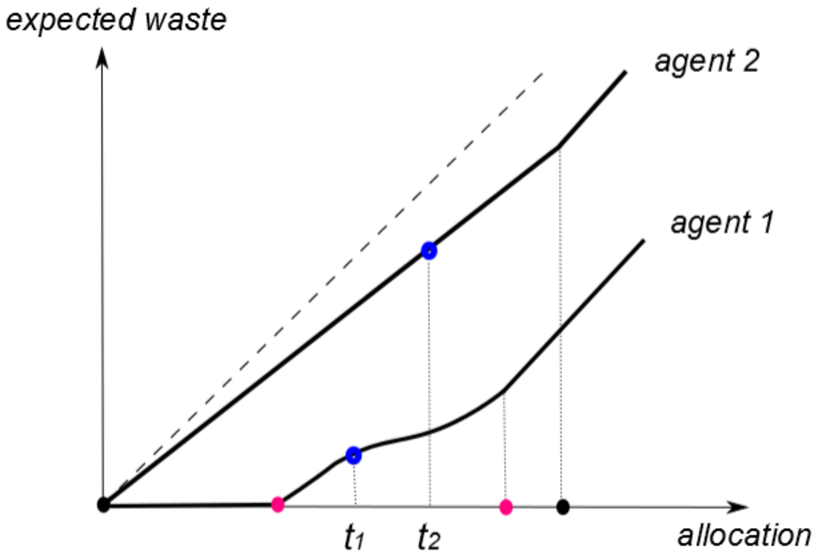
- Recourse monotonicity: Suppose that $F(I, (p_i)_{i \in I}, T) = t$ and $F(I, (p_i)_{i \in I}, T') = t'$. If $T < T'$, then $t_i \leq t'_i$ for all $i \in I$. Besides, the strict inequality holds if $t_i < \max \operatorname{supp}(p_i)$, $i \in I$.

Axioms

- Consistency: Suppose $F(I, (p_i)_{i \in I}, T) = t$. If $J \subseteq I$, then $F(J, (p_i)_{i \in J}, \sum_{i \in J} t_i) = t_J$.

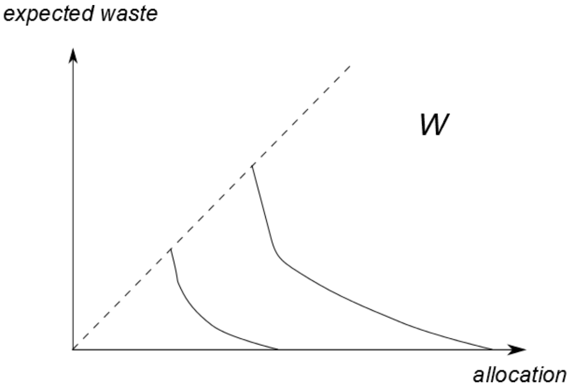
- Envy-freeness: Suppose $F(I, (p_i)_{i \in I}, T) = t$. If $t_i > t_j$ where $i, j \in I$, then either
  $t_j = \max \operatorname{supp}(p_j)$, or
  $\int_{t_i \geq x} (t_i - x) dp_i < \int_{t_j \geq x} (t_j - x) dp_j$.

An illustration of envy-freeness



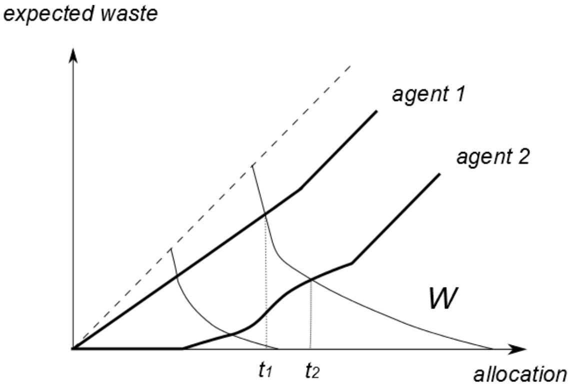
A generalized Uniform Gains rule

- Let $\mathcal{D} = \{(x, y) \in \mathbb{R} \mid x > y\} \cup \{(0, 0)\}$.
- We say a real-valued function W on $\mathcal{D}$ is increasing if $W(x, y) < W(x', y')$ when $x < x'$ and $y \leq y'$.



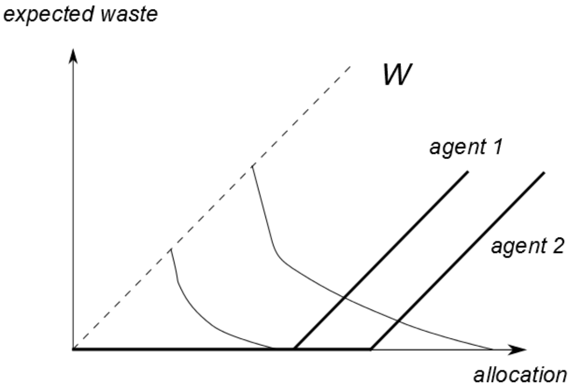
A generalized Uniform Gains rule

- Let $\mathcal{M} = \{F \mid$ there exists an increasing and continuous real-valued function W on $\mathcal{D}$ such that if $F(I, (p_i)_{i \in I}, T) = t$, then either $W(t_i, \int_{t_i \geq x} (t_i - x) dp_i) = \lambda$, or $t_i = \max \mathrm{supp}(p_i)$, $\forall i \in I\}$.



A generalized Uniform Gains rule

- $F \in \mathcal{M}$ generalizes the Uniform Gains division rule for deterministic demands.



The main result

A division rule F is anonymous, recourse monotonic, consistent and envy-free if and only if $F \in \mathcal{M}$.

Summary

- Extend the classic rationing problem to incorporate demand uncertainty.
- Characterize two family of division rules.
- Future work: concrete form of W and the parental preference over random demands.