Evolutionary Selection of Individual Expectations and Aggregate Outcomes in Asset Pricing Experiments

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Expectations in Economic Theory

- Economy is an expectation feedback system
  - Expectations affect our decisions and realizations
  - Expectations are affected by past experience

- Expectations play the key role in most economic models

  - 30s-60s naive and adaptive expectations
  - 70s-90s rational expectations
  - 90s- models of learning and bounded rationality
    - Adaptive learning (OLS-learning)
    - Belief-based learning
    - Reinforcement learning
Example 1: Model of Financial Market

- demand for the risky asset (available in zero supply)

\[ D_h(p_t) = \frac{E_{h,t}[p_{t+1} + y_{t+1}] - (1 + r)p_t}{a \ V_{h,t}[p_{t+1} + y_{t+1}]} \]

- solving market clearing eq. at time \( t \) find the equilibrium price

\[ \sum_h D_h(p_t) = 0 \implies p_t = \frac{1}{1 + r} \sum_h E_{h,t}[p_{t+1} + y_{t+1}] \]

- rational expectations

\[ p_t = \frac{1}{1 + r} E_t[p_{t+1} + y_{t+1}] \implies (\text{for i.i.d. dividends}) \ p_t = \frac{\bar{y}}{r} \]

- heterogeneous expectations (Brian Arthur, 1991)

\[ p_t = \frac{1}{1 + r} \sum_h E_{h,t} \left[ \frac{1}{1 + r} \sum_{h'} E_{h',t+1}[p_{t+2} + y_{t+2}] + y_{t+1} \right] \]
This paper:

▶ Heuristic Switching Model with heterogeneous expectations

▶ model is inspired by and tested on the experimental data

Key features:

in forecasting agents use simple rules of thumb, heuristics
   (Tversky and Kahneman, 1974)

in learning agents switch between different forecasting rules on the basis of their performances
   (Brock and Hommes, 1997)
Experiments about expectations

- Earlier experiments: indirect focus / expectations on exogenous time series: Schmalensee (1976), Hey (1994), Marimon and Sunder (1994)

Model of asset-pricing (Campbell, Lo and MacKinlay, 1997)

- riskless asset with interest $r = 0.05$
- risky asset with price $p_t$ and i.i.d. dividend $y_t$ with mean $\bar{y} = 3$

$$p_t = \frac{1}{1+r} \left( \bar{p}_{t+1}^e + \bar{y} + \varepsilon_t \right) = \frac{1}{1+r} \left( \frac{p_{t+1,1}^e + \cdots + p_{t+1,6}^e}{6} + \bar{y} + \varepsilon_t \right)$$

Experiment: 6 human subjects know only qualitative features

- submit forecasts $p_{t+1,h}^e$ and are paid according to the precision
- observe past prices (up to $p_{t-1}$), own forecasts and payoffs
earnings per period: \[ e_{t,h} = \max \left( 1 - \frac{1}{49} (p_t - p_{t,h}^e)^2, 0 \right) \times \frac{1}{2} \text{ euro} \]
Rational Benchmark

If everybody predicts fundamental price \( p_f = \frac{\bar{y}}{r} = 60 \), then \( p_t = p_f + \frac{\varepsilon_t}{1+r} \)
Evolutionary Selection of Individual Expectations
Experiment with stabilizing fundamentalists

- pricing equation

\[ p_t = \frac{1}{1+r} \left( (1 - n_t)\bar{p}^e_{t+1} + n_t p^f + \bar{y} + \varepsilon_t \right) \]

- fraction of fundamental traders

\[ n_t = 1 - \exp \left( - \frac{1}{200} |p_{t-1} - p^f| \right) \]
Evolutionary Selection of Individual Expectations
Results (individual predictions)
Estimation of individual prediction rules

OLS regression of predictions on the lagged prices and predictions

\[ p_{i,t+1}^e = \alpha + \sum_{k=1}^{5} \beta_k p_{t-k} + \sum_{k=0}^{5} \gamma_k p_{i,t-k} + \epsilon_{i,t} \]

leaving insignificant coefficients out

- adaptive expectations

\[ p_{t+1,h}^e = w p_{t-1} + (1 - w) p_{t,h}^e \]

- trend-extrapolating rules

\[ p_{t+1,h}^e = p_{t-1} + \gamma (p_{t-1} - p_{t-2}) \]

- anchoring and adjustment rule

\[ p_{t+1,h}^e = \frac{1}{2} (60 + p_{t-1}) + (p_{t-1} - p_{t-2}) \]
Learning-to-forecast experiments: Summary

“Stylized facts”

- large bubbles in the absence of fundamentalists
- qualitatively different patterns in the same environment
  - (almost) monotonic convergence
  - constant oscillations
  - damping oscillations
- coordination of individual predictions
- forecasting rules with behavioral interpretation are used
Model: four forecasting heuristics

▶ adaptive rule

ADA \[ p^e_{1,t+1} = 0.65 p_{t-1} + 0.35 p^e_{1,t} \]

▶ weak trend-following rule

WTR \[ p^e_{2,t+1} = p_{t-1} + 0.4 (p_{t-1} - p_{t-2}) \]

▶ strong trend-following rule

STR \[ p^e_{3,t+1} = p_{t-1} + 1.3 (p_{t-1} - p_{t-2}) \]

▶ anchoring and adjustment heuristics with learnable anchor

LAA \[ p^e_{4,t+1} = \frac{1}{2} (p^a_{t-1} + p_{t-1}) + (p_{t-1} - p_{t-2}) \]

price dynamics

\[ p_t = \frac{1}{1+r} \left( (n_1, t p^e_{1,t+1} + n_2, t p^e_{2,t+1} + n_3, t p^e_{3,t+1} + n_4, t p^e_{4,t+1}) + \bar{y} + \epsilon_t \right) \]
Adaptive Expectations: \( p_{t+1}^e = w p_{t-1} + (1 - w) p_t^e \)

Dynamics globally converge to fundamental price.
Weak-Trend Extrapolation: \( p_{t+1}^e = p_{t-1} + \gamma (p_{t-1} - p_{t-2}) \)

Dynamics converge to fundamental price.
Strong-Trend Extrapolation: $p_{t+1}^e = p_{t-1} + \gamma (p_{t-1} - p_{t-2})$

Dynamics **diverge** from fundamental price...
Strong-Trend Extrapolation: $p_{t+1}^e = p_{t-1} + \gamma (p_{t-1} - p_{t-2})$

...and settles on the quasi-periodic attractor.

Attractor under strong trend following heuristic

\[ \gamma = 1.1 \]  \[ \gamma = 1.3 \]

200 points after 500 transitory steps
Anchoring and Adjustment: $p_{t+1}^e = \frac{p_f^t + p_{t-1}}{2} + (p_{t-1} - p_{t-2})$

Price under anchoring and adjustment heuristic

with learning of anchor: $p_{t+1}^e = \frac{p_{t-1}^av + p_{t-1}}{2} + (p_{t-1} - p_{t-2})$
Model with Homogeneous Expectations

- Pattern of **monotonic convergence** can be easily reproduced with an adaptive rule, weak trend extrapolation.

- Pattern of **constant oscillations** can be reproduced with anchoring and adjustment rule without learning.

- Pattern of **damping oscillations** is reproduced (very imperfectly) for strong-trend extrapolations.
Stability conditions

\[ p_{t+1}^e = \alpha + \beta_1 p_{t-1} + \beta_2 p_{t-2} \]
Evidence for switching I

Group 6, participant 1

prediction ---
price -----

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Evolutionary Selection of Individual Expectations
Evidence for switching II

Group 1, participant 3

prediction

price

Time
Modelling switching behavior

impacts of heuristics $n_{i,t}$ are evolving

- performance measure of heuristic $i$ is

$$U_{i,t-1} = - (p_{t-1} - p_{i,t-1}^e)^2 + \eta U_{i,t-2}$$

parameter $\eta \in [0, 1]$ – the strength of the agents’ memory

- discrete choice model with asynchronous updating

$$n_{i,t} = \delta n_{i,t-1} + (1 - \delta) \frac{\exp(\beta U_{i,t-1})}{\sum_{i=1}^{4} \exp(\beta U_{i,t-1})}$$

parameter $\delta \in [0, 1]$ – the inertia of the traders

parameter $\beta \geq 0$ – the intensity of choice
Stability and Instability of the model

Stability region for model with fixed fractions

\[ n_1, \text{fraction of ADA} \]
\[ n_2, \text{fraction of WTR} \]

\[ n_3, \text{fraction of STR} \]
Deterministic path: Monotonic convergence

Parameters: $\beta = 0.4, \eta = 0.7, \delta = 0.9$
Deterministic path: Constant oscillations

Parameters: \( \beta = 0.4, \eta = 0.7, \delta = 0.9 \)
Deterministic path: Damping oscillations

Parameters: \( \beta = 0.4, \eta = 0.7, \delta = 0.9 \)
Evolution of instability

Largest modulus of eigenvalue of HSM
One-period ahead prediction: gr. 6 (constant oscillations)

Parameters: $\beta = 0.4$, $\eta = 0.7$, $\delta = 0.9$
One-period ahead prediction: gr. 4 (damping oscillations)

Parameters: $\beta = 0.4, \eta = 0.7, \delta = 0.9$
One-period ahead prediction: gr. 5 (convergence)

Parameters: $\beta = 0.4, \eta = 0.7, \delta = 0.9$
In-sample performance
MSE over 47 periods for 8 different models

<table>
<thead>
<tr>
<th>Specification</th>
<th>Group 2</th>
<th>Group 5</th>
<th>Group 1</th>
<th>Group 6</th>
<th>Group 4</th>
<th>Group 7</th>
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</thead>
<tbody>
<tr>
<td>Fundamental</td>
<td>16.6231</td>
<td>10.8238</td>
<td>15.7581</td>
<td>9.3245</td>
<td>300.9936</td>
<td>21.9123</td>
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<td>ADA</td>
<td>0.0712</td>
<td>0.0378</td>
<td>5.6734</td>
<td>4.6095</td>
<td>210.3313</td>
<td>19.5158</td>
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<td>WTR</td>
<td>0.0862</td>
<td>0.1419</td>
<td>2.0905</td>
<td>1.1339</td>
<td>92.2163</td>
<td>9.2932</td>
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<td>STR</td>
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<td>0.6605</td>
<td>2.9071</td>
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<td>LAA</td>
<td>0.4588</td>
<td>0.4756</td>
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<td>0.6591</td>
<td>66.2637</td>
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<td>HSM</td>
<td>0.0814</td>
<td>0.1698</td>
<td>1.2417</td>
<td>0.6618</td>
<td>70.8516</td>
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<td>HSM (fitted)</td>
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<tr>
<td>$\beta \in [0, 10]$</td>
<td>10</td>
<td>10</td>
<td>0.1</td>
<td>10</td>
<td>3</td>
<td>0.2</td>
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<tr>
<td>$\eta \in [0, 1]$</td>
<td>0.4</td>
<td>0.9</td>
<td>1</td>
<td>0.1</td>
<td>0.8</td>
<td>0.5</td>
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<tr>
<td>$\delta \in [0, 1]$</td>
<td>0.9</td>
<td>0.6</td>
<td>0.5</td>
<td>0.7</td>
<td>0.6</td>
<td>0.4</td>
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Out-of-sample performance

The Heuristic Switching Model vs. AR(2) model

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<tr>
<td><strong>HSM</strong></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>1 p ahead</td>
<td>0.0122</td>
<td>0.0321</td>
<td>0.479</td>
<td>0.1921</td>
<td>15.0395</td>
<td>0.7857</td>
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<td>2 p ahead</td>
<td>0.0122</td>
<td>0.0901</td>
<td>1.8599</td>
<td>1.0792</td>
<td>57.5144</td>
<td>1.5543</td>
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<td><strong>AR(2) Model</strong></td>
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<tr>
<td>1 p ahead</td>
<td>0.3732</td>
<td>0.4431</td>
<td>0.9981</td>
<td>0.5568</td>
<td>13.4616</td>
<td>0.6682</td>
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<tr>
<td>2 p ahead</td>
<td>0.5052</td>
<td>0.4045</td>
<td>3.5823</td>
<td>1.4944</td>
<td><strong>44.6453</strong></td>
<td>2.0098</td>
</tr>
</tbody>
</table>
Conclusion

- the model with evolutionary switching between simple heuristics
- dynamics of the model is path-dependent (different patterns of the experiments have been reproduced)
- good in-sample and out-of-sample performance
- Anufriev and Hommes (2012, AEJ-Micro and 2012, KER)

Model applications

- macroeconomics
- financial bubbles and crashes
- agent-based modelling
Conclusion

- the model with evolutionary switching between simple heuristics
- dynamics of the model is path-dependent (different patterns of the experiments have been reproduced)
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Model applications

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Further Work

- theoretical relation of an outcome with Restricted Perception Equilibrium
- further analysis (other experiments, other methods)
- comparison with other learning methods (e.g., with GA)
- direct experiment on switching to estimate switching parameters
- classification of behavioral types on the basis of individual predictions
Learning in a Complex Environment

Mailath (JEL, 1998) *Do people play Nash equilibrium? Lessons from evolutionary game theory*, p. 1349-1350:

The typical agent is not like Gary Kasparov, the world champion chess player who knows the rules of chess, but also knows that he doesn’t know the winning strategy.

In most situations, people do not know they are playing a game. Rather, people have some (perhaps imprecise) notion of the environment they are in, their possible opponents, the actions they and their opponents have available, and the possible payoff implications of different actions.

These people use heuristics and rules of thumb (generated from experience) to guide behavior; sometimes these heuristics work well and sometimes they don’t. These heuristics can generate behavior that is inconsistent with straightforward maximization.
Dynamics for Individual Rules: Converging Groups

\[ p_{t+1}^e = \alpha + \beta_1 p_{t-1} + \beta_2 p_{t-2} \]
Dynamics for Individual Rules: Oscillating Groups

\[ p_{t+1}^e = \alpha + \beta_1 p_{t-1} + \beta_2 p_{t-2} \]
Dynamics for Individual Rules: Damping Groups

\[ p_{t+1}^e = \alpha + \beta_1 p_{t-1} + \beta_2 p_{t-2} \]
Stability for the Model with **Fixed Impacts**

Stability region for model with fixed fractions

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Evolutionary Selection of Individual Expectations
Comparison with Homogeneous Expectations: MSE

“Direct fit” with parameters $\beta = 0.4$, $\eta = 0.7$, $\delta = 0.9$

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</thead>
<tbody>
<tr>
<td>Fundamental Prediction</td>
<td>18.037</td>
<td>11.797</td>
<td>15.226</td>
<td>8.959</td>
<td>291.376</td>
<td>22.047</td>
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<tr>
<td>ADA – exp prices</td>
<td>0.841</td>
<td>0.200</td>
<td>7.676</td>
<td>8.401</td>
<td>330.101</td>
<td>51.526</td>
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<tr>
<td>WTR – exp prices</td>
<td>4.419</td>
<td>1.983</td>
<td>8.868</td>
<td>6.252</td>
<td>308.549</td>
<td>30.298</td>
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<tr>
<td>STR – exp prices</td>
<td>585.789</td>
<td>478.525</td>
<td>638.344</td>
<td>509.266</td>
<td>1231.064</td>
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<tr>
<td>LAA – exp prices</td>
<td>5.475</td>
<td>3.534</td>
<td>5.405</td>
<td>14.404</td>
<td>307.605</td>
<td>69.749</td>
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<tr>
<td>ADA – fitted prices</td>
<td>0.514</td>
<td>0.199</td>
<td>6.832</td>
<td>7.431</td>
<td>312.564</td>
<td>36.436</td>
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<td>WTR – fitted prices</td>
<td>4.222</td>
<td>1.844</td>
<td>8.670</td>
<td>6.228</td>
<td>292.150</td>
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<td>STR – fitted prices</td>
<td>413.435</td>
<td>42.488</td>
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<td>4 heuristics (plots)</td>
<td>0.449</td>
<td>0.302</td>
<td>8.627</td>
<td>14.755</td>
<td>526.417</td>
<td>29.520</td>
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<tr>
<td>4 heuristics (fitted)</td>
<td>0.313</td>
<td>0.245</td>
<td>7.227</td>
<td>7.679</td>
<td>235.900</td>
<td>18.662</td>
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Comparison with Homogeneous Expectations: AR2

“Indirect fit” with parameters $\beta = 0.4, \eta = 0.7, \delta = 0.9$

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</thead>
<tbody>
<tr>
<td>Fundamental Prediction</td>
<td>0.946</td>
<td>0.671</td>
<td>2.673</td>
<td>3.610</td>
<td>2.311</td>
<td>2.002</td>
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<td>ADA – exp prices</td>
<td>0.239</td>
<td>0.006</td>
<td>2.182</td>
<td>2.898</td>
<td>1.691</td>
<td>1.494</td>
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<td>WTR – exp prices</td>
<td>0.066</td>
<td>0.529</td>
<td>0.383</td>
<td>0.627</td>
<td>0.203</td>
<td>0.165</td>
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<td>STR – exp prices</td>
<td>1.494</td>
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<td>AA – exp prices</td>
<td>1.095</td>
<td>1.848</td>
<td>0.010</td>
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<td>0.045</td>
<td>0.094</td>
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<td>0.747</td>
<td>1.544</td>
<td>0.003</td>
<td>0.050</td>
<td>0.003</td>
<td>0.013</td>
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<td>ADA – fitted prices</td>
<td>0.100</td>
<td>0.000</td>
<td>1.584</td>
<td>2.159</td>
<td>1.385</td>
<td>1.157</td>
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<tr>
<td>WTR – fitted prices</td>
<td>0.068</td>
<td>0.343</td>
<td>0.262</td>
<td>0.435</td>
<td>0.174</td>
<td>0.139</td>
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<tr>
<td>STR – fitted prices</td>
<td>1.358</td>
<td>2.192</td>
<td>0.078</td>
<td>0.001</td>
<td>0.147</td>
<td>0.242</td>
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<tr>
<td>AA – fitted prices</td>
<td>1.036</td>
<td>1.755</td>
<td>0.005</td>
<td>0.029</td>
<td>0.038</td>
<td>0.083</td>
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<td>LAA – fitted prices</td>
<td>0.640</td>
<td>1.277</td>
<td>0.000</td>
<td>0.033</td>
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<td>0.004</td>
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<td>4 heuristics (plots)</td>
<td>0.383</td>
<td>0.744</td>
<td>0.011</td>
<td>0.008</td>
<td>0.157</td>
<td>0.239</td>
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<td>4 heuristics (fitted)</td>
<td>0.144</td>
<td>0.499</td>
<td>0.009</td>
<td>0.003</td>
<td>0.121</td>
<td>0.048</td>
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</table>
Measure of coordination

- average prediction error = average dispersion + common prediction error

- in the experiment

\[
\frac{1}{6} \sum_{i=1}^{6} (p_{i,t}^e - p_t)^2 = \frac{1}{6} \sum_{i=1}^{6} (p_{i,t}^e - \bar{p}_t^e)^2 + (\bar{p}_t^e - p_t)^2,
\]

where \( \bar{p}_t^e = \frac{1}{6} \sum_{i=1}^{6} p_{i,t}^e \)

- in simulations

\[
\sum_{h=1}^{4} n_{h,t-1} (p_{h,t}^e - p_t)^2 = \sum_{h=1}^{4} n_{h,t-1} (p_{h,t}^e - \bar{p}_t^e)^2 + (\bar{p}_t^e - p_t)^2,
\]

where \( \bar{p}_t^e = \sum_{h=1}^{4} n_{h,t-1} p_{h,t}^e \)
Coordination

**Table:** Coordination in the experiment and over the simulations.

<table>
<thead>
<tr>
<th>period</th>
<th>Group 5</th>
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The same experiment: group 3

Parameters: $\beta = 0.4$, $\eta = 0.7$, $\delta = 0.9$
Other Experiments

\[ p_t = \frac{1}{1+r} \left( \bar{p}_{t+1}^e + \bar{y} + \varepsilon_t \right) \]
Bubbles in Experiment I

\[ p_t = \frac{1}{1+r} (\bar{p}_{t+1} + \bar{y}) \]
Bubbles in Experiment II

\[ p_t = \frac{1}{1+r}(\bar{p}_{t+1} + \bar{y}) \]
Other Experiments: Smaller Fundamental Price

\[ \bar{y} = 3 \rightarrow \bar{y} = 2 \]
Other Experiments: One-period ahead forecast

\[ p_t = \frac{1}{1+r} \left( p_{t+1}^{AE} + \bar{y} \right) \rightarrow p_t = \frac{1}{1+r} \left( p_t^{AE} + \bar{y} + \varepsilon_t \right) \]
Other Experiments: Negative Feedback

\[ p_t = a + b p_{t+1}^{AE} + \varepsilon_t \quad \rightarrow \quad p_t = a' - b p_{t+1}^{AE} + \varepsilon_t \]
Other Experiments: Smaller Fundamental Price I

\[
\bar{y} = 3 \rightarrow \bar{y} = 2
\]
Other Experiments: Smaller Fundamental Price II

\[ \bar{y} = 3 \rightarrow \bar{y} = 2 \]
Other Experiments: No Robots

\[ p_t = \frac{1}{1+r} \left( (1 - n_t)p_{t+1}^{AE} + n_t p^f + \bar{y} + \varepsilon_t \right) \rightarrow p_t = \frac{1}{1+r} \left( p_{t+1}^{AE} + \bar{y} \right) \]
Other Experiments: One-period ahead forecast

\[ p_t = \frac{1}{1+r} \left( p_{t+1}^{AE} + \bar{y} \right) \rightarrow p_t = \frac{1}{1+r} \left( p_t^{AE} + \bar{y} + \epsilon_t \right) \]
Other Experiments: Negative Feedback

\[ p_t = a + bp_{t+1}^{AE} + \varepsilon_t \quad \rightarrow \quad p_t = a' - bp_{t+1}^{AE} + \varepsilon_t \]