Unanimity Overruled: Majority Voting and the Burden of History

Clemens Puppe

joint work with Klaus Nehring and Marcus Pivato

Centre of Mathematical Social Sciences
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Motivation

Example (Sequential Majority Voting in Preference Aggregation)

Consider four alternatives $a$, $b$, $c$, $d$ and suppose that $\frac{1}{3}$ of the population endorses the preference orderings $a \succ_1 b \succ_1 c \succ_1 d$, $b \succ_2 c \succ_2 d \succ_2 a$ and $c \succ_3 d \succ_3 a \succ_3 b$, respectively.

'Condorcet paradox:' pairwise majority voting yields intransitivity. Sequential pairwise majority voting plus transitivity? May force one to override unanimous consent! E.g., if votes are cast in the order $(d, a, b, c)$ one obtains $d \succ a \succ b \succ c$, hence $d \succ c$ by transitivity, although there is unanimous consent that $c$ is better than $d$. 
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Consider four alternatives $a$, $b$, $c$, $d$ and suppose that $\frac{3}{4}$ of the population endorses the preference orderings $a \succ_1 b \succ_1 c \succ_1 d$, $b \succ_2 c \succ_2 d \succ_2 a$ and $c \succ_3 d \succ_3 a \succ_3 b$, respectively. 'Condorcet paradox:' pairwise majority voting yields intransitivity. Sequential pairwise majority voting plus transitivity? May force one to override unanimous consent! E.g., if votes are cast in the order $(d, a, b, c)$ one obtains $d \succ a \succ b \succ c$, hence $d \succ c$ by transitivity, although there is unanimous consent that $c$ is better than $d$. 

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Questions

Why can the problem not occur with three alternatives only?

How general is the phenomenon?

Does it apply to judgment aggregation in general?

Can the problem be avoided by an appropriate choice of a decision sequence?
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1. Sequential Majority Voting
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   - The Judgement Aggregation Problem
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   - Characterization of Path-Independence
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   - Sequential Majority Voting and the Condorcet Set
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2. Path-Dependence and Unanimity Violations
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   - Strong Sequential Unanimity Consistency
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The Judgement Aggregation Problem

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The Judgement Aggregation Problem

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Path-(In)dependence

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The Judgement Aggregation Problem

Sequential majority voting is path-independent given $\mu$, that is:

$$\text{SMV}_\gamma(\mu) = \text{SMV}_\delta(\mu)$$

for all paths $\gamma, \delta$, if and only if the issue-wise majority view given $\mu$ is feasible.

Example (Preference Aggregation):

Strict orderings over alternatives $a, b, c$.

Issue 1: $a \succ b$?, issue 2: $b \succ c$?, issue 3: $c \succ a$?

Thus, $X_{\text{pref}} = \{0, 1\}^3\{0, 1, 0\}$.

The issue-wise majority view may be infeasible: E.g. 1/3 of the population endorse $(1, 1, 0)$ ["$a \succ b \succ c$"], 1/3 endorse $(0, 1, 1)$ ["$b \succ c \succ a$"], and another 1/3 endorse $(1, 0, 1)$ ["$c \succ a \succ b$"], then issue-wise majority view $(1, 1, 1) \not\in X_{\text{pref}}$.

SMV yields either $(1, 1, 0)$, $(0, 1, 1)$, or $(1, 0, 1)$. 

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- SMV yields either \( (1, 1, 0), (0, 1, 1), \) or \( (1, 0, 1) \).

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Examples (cont.)

Example (Resource Allocation):
Budget $L$ to be spent on $M$ public goods.

Issues: "spend at least $\ell$ dollars for good $m$?"
with feasibility constraint that exactly $L$ dollars spent in total.

E.g. 1/3 of the population endorse $(L-2, 1, 1)$,
1/3 endorse $(1, L-1, 0)$, and 1/3 endorse $(0, 0, L)$.

Then, majority view $(1, 1, 1) \not\in X$ if $L > 3$.

Observe that issue-wise majority view equals coordinate-wise median.

Outcomes of SMV:

$X^2 = (0, 0, L)$

$X^1 = (L, 0, 0)$

$X^3 = \text{coordinate-wise median}$
Examples (cont.)

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  - Budget $L$ to be spent on $M$ public goods.
  - Issues: "spend at least $\ell$ dollars for good $m$?"
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Outcomes of SMV:

- $(0, 0, L)$
- $(L, 0, 0)$
- $x_2$
- $x_1$
- $x_3$
- coordinate-wise median

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Examples (cont.)

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Outcomes of SMV:
More Examples

Example (Committee Selection):

$K$ candidates for election into a committee with at least $I$ members ($I \leq K$) and at most $J$ members ($I \leq J \leq K$).

Issues: "elect candidate $k$?"

Again, feasibility problem arises:

E.g. 1 of the population endorses each of $(1, 0, 1, 0)$, $(0, 1, 1, 0)$, and $(0, 0, 1, 1)$, respectively.

Then, if $I = J = 2$, issue-wise majority view $(0, 0, 1, 0) \not\in X_{com}$.

If $I = J = 2$, SMV elects candidate 3 plus any one of the other candidates.

Further examples: aggregation of weak orders, equivalence relations, partial orders, group identification à la Kasher and Rubinstein, reason based choice in legal contexts (the "doctrinal paradox"), probability aggregation, etc.
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Sequential Majority Voting

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Characterization of Path-Indepedence

When is SMV Path-Independent for all Profiles?

Definition

A forbidden fragment of length $k \leq K$ is a collection of judgements on a subset of $k$ issues that cannot be extended to a feasible view on $X$.

A forbidden fragment is called critical if it does not contain a strictly smaller forbidden fragment.

Theorem (NP 2002/2007)

Issue-wise majority voting is feasible for all profiles of feasible views if and only if all critical fragments of $X$ have length $\leq 2$.

Corollary

SMV is path-independent for all profiles of feasible views if and only if all critical fragments of $X$ have length $\leq 2$. 
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Sequential Majority Voting and the Condorcet Set

Agenda

1. Sequential Majority Voting
   - The Judgement Aggregation Problem
   - Characterization of Path-Independent
   - Sequential Majority Voting and the Condorcet Set

2. Path-Dependence and Unanimity Violations
   - Strong Sequential Unanimity Consistency
   - Weak Sequential Unanimity Consistency

3. Conclusion
Sequential Majority Voting and the Condorcet Set

The Condorcet Set (Nehring, Pivato and Puppe 2011)
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Definition

Given a profile $\mu \in X^N$ of feasible views, the Condorcet set $\text{Cond}(\mu) \subseteq X$ is the set of all $x \in X$ such that no feasible view coincides with the issue-wise majority view on a strictly larger set of issues than $x$. 
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Proposition

For all $X$ and all $\mu$, the Condorcet set coincides with the set of outcomes of sequential majority voting:
Sequential Majority Voting

Sequential Majority Voting and the Condorcet Set

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**Proposition**

For all $X$ and all $\mu$, the Condorcet set coincides with the set of outcomes of sequential majority voting:

$$x \in \text{Cond}(\mu) \iff x = \text{SMV}_\gamma(\mu) \text{ for some path } \gamma.$$
Sequential Majority Voting and the Condorcet Set

Example (Preference Aggregation)

As above, consider a, b, c, d and suppose that 1 of the population endorses the preference orderings a ≻ b ≻ c ≻ d, b ≻ c ≻ d ≻ a and c ≻ d ≻ a ≻ b, respectively.

The Condorcet admissible set consists of the following five orderings:

a ≻ b ≻ c ≻ d, b ≻ c ≻ d ≻ a, c ≻ d ≻ a ≻ b, d ≻ a ≻ b ≻ c, c ≻ a ≻ b ≻ d.
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As above, consider $a, b, c, d$ and suppose that $\frac{1}{3}$ of the population endorses the preference orderings $a \succ_1 b \succ_1 c \succ_1 d$, $b \succ_2 c \succ_2 d \succ_2 a$ and $c \succ_3 d \succ_3 a \succ_3 b$, respectively.
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- $b \succ c \succ d \succ a$, 
- $c \succ d \succ a \succ b$, 
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![Diagram showing the Condorcet admissible set with arrows between $a$, $b$, $c$, and $d$.]
Sequential Majority Voting and the Condorcet Set

Example (Resource Allocation)

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Sequential Majority Voting and the Condorcet Set

Example (Resource Allocation)

Consider $X_{L,M}^{alloc}$ and denote by $y^m$ the amount spent on good $m$. Given profile $\mu$, let $\text{med}_m(\mu)$ be the median amount proposed for good $m$ and $D(\mu) := \left(\sum_{m=1}^{M} \text{med}_m(\mu) - L\right)$ the 'majority deficit.' The Condorcet set is given as follows:

If $D(\mu) \geq 0$, then

$$\text{Cond}(\mu) = \left\{ y \in X_{L,M}^{alloc} : y^m \in [\text{med}_m(\mu) - D(\mu), \text{med}_m(\mu)] \forall m \right\}$$

if $D(\mu) \leq 0$, then

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Example (Committee Selection)

Consider $X = \{I, J, K\}$, and suppose that $Q \subseteq \{1, \ldots, K\}$ is the set of candidates that receive majority support under the profile $\mu$. The Condorcet set is given as follows:

- If $I \leq \#Q \leq J$, then $\text{Cond}(\mu) = \{1\}^Q$.
- If $\#Q < I$, then $\text{Cond}(\mu) = \{1\}^H: Q \subset H$ and $\#H = I$.
- If $J < \#Q$, then $\text{Cond}(\mu) = \{1\}^H: H \subset Q$ and $\#H = J$. 
Example (Committee Selection)

Consider $X_{I,J;K}^{\text{com}}$, and suppose that $Q \subseteq \{1, \ldots, K\}$ is the set of candidates that receive majority support under the profile $\mu$. 
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Consider \( X_{I,J,K}^{\text{com}} \), and suppose that \( Q \subseteq \{1, \ldots, K\} \) is the set of candidates that receive majority support under the profile \( \mu \). The Condorcet set is given as follows:
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Consider $X_{I,J;K}^{com}$, and suppose that $Q \subseteq \{1, ..., K\}$ is the set of candidates that receive majority support under the profile $\mu$. The Condorcet set is given as follows:

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Sequential Majority Voting

Path-Dependence and Unanimity Violations

Conclusion

Sequential Majority Voting and the Condorcet Set

Example (Committee Selection)

Consider $X^\text{com}_{I,J;K}$, and suppose that $Q \subseteq \{1, \ldots, K\}$ is the set of candidates that receive majority support under the profile $\mu$. The Condorcet set is given as follows:

If $I \leq \#Q \leq J$, then $\text{Cond}(\mu) = \{1_Q\}$,

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Sequential Majority Voting and the Condorcet Set

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Unanimity Overruled: Majority Voting and the Burden of History
Strong Sequential Unanimity Consistency

Definition and General Characterization

A space $X$ is strongly sequentially unanimity consistent if, for no path $\gamma$ and for no profile $\mu$, SMV$_\gamma(\mu)$ overrides a unanimous judgement in any issue.

Theorem

A space $X$ is strongly sequentially unanimity consistent if and only if all critical fragments of $X$ have length $\leq 3$. 

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Unanimity Overruled: Majority Voting and the Burden of History
Strong Sequential Unanimity Consistency

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Examples

Let \( X_{\text{pref}}^q \) denote the space of all linear preference orderings over \( q \) alternatives. Then, there exist critical fragments of all lengths up to \( q \). Hence, \( X_{\text{pref}}^q \) is strongly sequentially unanimity consistent if and only if \( q \leq 3 \). The spaces \( X_{\text{alloc}}^L, M \) are strongly sequentially unanimity consistent if and only if \( M \leq 3 \). One can show that the longest critical fragments in \( X_{\text{com}}^I, J; K \) have length \( 1 + \max\{J, K - I\} \). Hence, \( X_{\text{com}}^I, J; K \) is not strongly sequentially unanimity consistent whenever \( K \geq 5 \). On the other hand, e.g., \( X_{\text{com}}^2, 2; 4 \) is strongly sequentially unanimity consistent.
Examples

- Let $X_q^{\text{pref}}$ denote the space of all linear preference orderings over $q$ alternatives.
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- Let $X_q^{\text{pref}}$ denote the space of all linear preference orderings over $q$ alternatives. Then, there exist critical fragments of all lengths up to $q$. Hence, $X_q^{\text{pref}}$ is strongly sequentially unanimity consistent if and only if $q \leq 3$.

- The spaces $X_{L,M}^{\text{alloc}}$ are strongly sequentially unanimity consistent if and only if $M \leq 3$. 
Strong Sequential Unanimity Consistency

Examples

- Let $X^\text{pref}_q$ denote the space of all linear preference orderings over $q$ alternatives. Then, there exist critical fragments of all lengths up to $q$. Hence, $X^\text{pref}_q$ is strongly sequentially unanimity consistent if and only if $q \leq 3$.

- The spaces $X^\text{alloc}_{L,M}$ are strongly sequentially unanimity consistent if and only if $M \leq 3$.

- One can show that the longest critical fragments in $X^\text{com}_{I,J;K}$ have length $1 + \max\{J, K-I\}$.
Strong Sequential Unanimity Consistency

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- Let $X_q^{\text{pref}}$ denote the space of all linear preference orderings over $q$ alternatives. Then, there exist critical fragments of all lengths up to $q$. Hence, $X_q^{\text{pref}}$ is strongly sequentially unanimity consistent if and only if $q \leq 3$.

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- Let $X_q^{\text{pref}}$ denote the space of all linear preference orderings over $q$ alternatives. Then, there exist critical fragments of all lengths up to $q$. Hence, $X_q^{\text{pref}}$ is strongly sequentially unanimity consistent if and only if $q \leq 3$.

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Hence, $X_{l,j,k}^{\text{com}}$ is not strongly sequentially unanimity consistent whenever $k \geq 5$. 
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- One can show that the longest critical fragments in $X_{I,J,K}^{\text{com}}$ have length

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Hence, $X_{I,J,K}^{\text{com}}$ is not strongly sequentially unanimity consistent whenever $K \geq 5$. On the other hand, e.g., $X_{2,2,4}^{\text{com}}$ is strongly sequentially unanimity consistent.
Sequential Majority Voting

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Characterization of Path-Independence
Sequential Majority Voting and the Condorcet Set

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Conclusion

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Unanimity Overruled: Majority Voting and the Burden of History
Weak Sequential Unanimity Consistency

**Definition and Examples**

**Definition**

A space $X$ is weakly sequentially unanimity consistent if there exists a path $\gamma$ such that for no profile $\mu$, $\text{SMV}_\gamma(\mu)$ overrides a unanimous judgement in any issue.

**Proposition**

The spaces $X_{\text{alloc}}$ and $X_{\text{com}}$ are weakly sequentially unanimity consistent if and only if they are even strongly sequentially unanimous consistent.
Weak Sequential Unanimity Consistency

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The spaces $X^{\text{alloc}}$ and $X^{\text{com}}$ are weakly sequentially unanimity consistent if and only if they are even strongly sequentially unanimously consistent.
The Weak Sequential Unanimity Consistency of $X^{\text{pref}}$

Theorem (adapted from Shepsle and Weingast 1984)

The spaces $X^{\text{pref}}$ are weakly sequentially unanimity consistent.

Idea of proof:

Let $\succ^{\mu}$ denote the majority tournament given $\mu$.

Define the corresponding 'covering relation' by $a \succ^{\ast} \mu b \iff [a \succ^{\mu} b \land \forall c ((b \succ^{\mu} c \Rightarrow a \succ^{\mu} c) \land \forall c ((c \succ^{\mu} a \Rightarrow c \succ^{\mu} b))]$.

$\succ^{\ast} \mu$ is transitive and extends the unanimity relation.

Identify the alternatives with 1, 2, 3, ..., $q$ and define a path $\zeta$ by $(1, 2), (1, 3), (1, 4), \ldots, (1, q), (2, 3), (2, 4), \ldots, (q-1, q)$.

Show that $\text{SMV}_\zeta(\mu)$ extends $\succ^{\ast} \mu$.
The Weak Sequential Unanimity Consistency of $X^{pref}$

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The Weak Sequential Unanimity Consistency of $X^{\text{pref}}$

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The Weak Sequential Unanimity Consistency of $X^{\text{pref}}$

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*The spaces $X_q^{\text{pref}}$ are weakly sequentially unanimity consistent.*

**Idea of proof:**
- Let $\succ_\mu$ denote the majority tournament given $\mu$.
- Define the corresponding ‘covering relation’ by

$$ a \succ_\mu^* b \iff [a \succ_\mu b \text{ and for all } c, (b \succ_\mu c \Rightarrow a \succ_\mu c) \& (c \succ_\mu a \Rightarrow c \succ_\mu b)]. $$

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- $\succ^*_\mu$ is transitive and extends the unanimity relation.
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- $\succ^*_\mu$ is transitive and extends the unanimity relation.
- Identify the alternatives with 1, 2, 3, ..., $q$ and define a path $\zeta$. 

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Unanimity Overruled: Majority Voting and the Burden of History
The Weak Sequential Unanimity Consistency of $X_{q}^{\text{pref}}$

**Theorem (adapted from Shepsle and Weingast 1984)**

The spaces $X_{q}^{\text{pref}}$ are weakly sequentially unanimity consistent.

**Idea of proof:**
- Let $\succ_{\mu}$ denote the majority tournament given $\mu$.
- Define the corresponding ‘covering relation’ by
  
  \[ a \succ^{\ast}_{\mu} b \Leftrightarrow [a \succ_{\mu} b \text{ and for all } c, (b \succ_{\mu} c \Rightarrow a \succ_{\mu} c) \& (c \succ_{\mu} a \Rightarrow c \succ_{\mu} b)] . \]

- $\succ^{\ast}_{\mu}$ is transitive and extends the unanimity relation.
- Identify the alternatives with 1, 2, 3, …, $q$ and define a path $\zeta$ by $(1, 2), (1, 3), (1, 4), \ldots, (1, q), (2, 3), (2, 4), \ldots, (3, 4), \ldots, (q - 1, q)$. 

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  $(1, 2), (1, 3), (1, 4), ...., (1, q), (2, 3), (2, 4), ...., (3, 4), ...., (q - 1, q)$.

- Show that $\text{SMV}_\zeta(\mu)$ extends $\succ^* \mu$. 
Generalization to ‘Simple Spaces’ of Transitive Relations

A space $X$ is a simple space of transitive relations if all critical fragments are entailed either by transitivity, symmetry, or asymmetry restrictions, respectively.

Examples of simple spaces of transitive relations are the spaces of all linear orders, all weak orders, all strict partial orders, all weak partial orders, and all equivalence relations.

Theorem

All simple spaces of transitive relations are weakly sequentially unanimity consistent.
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All simple spaces of transitive relations are weakly sequentially unanimity consistent.
Agenda

1. Sequential Majority Voting
   - The Judgement Aggregation Problem
   - Characterization of Path-Indepedence
   - Sequential Majority Voting and the Condorcet Set

2. Path-Dependence and Unanimity Violations
   - Strong Sequential Unanimity Consistency
   - Weak Sequential Unanimity Consistency

3. Conclusion
Concluding Remarks
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- We have characterized all spaces which are strongly sequentially unanimity consistent, i.e. in which sequential majority voting never overrides unanimous consent, no matter in which sequence the voting takes place.
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- The characterizing condition is a simple generalization of the condition that is necessary and sufficient for the consistency of issue-wise majority voting (given any profile of individual views).
Concluding Remarks

- We have characterized all spaces which are strongly sequentially unanimity consistent, i.e. in which sequential majority voting never overrides unanimous consent, no matter in which sequence the voting takes place.
- The characterizing condition is a simple generalization of the condition that is necessary and sufficient for the consistency of issue-wise majority voting (given any profile of individual views).
- Very few aggregation problems verify this condition.
Concluding Remarks

Remarkably, some important aggregation problems that are not strongly sequentially unanimity consistent satisfy the weaker requirement that there exists some decision path along which unanimous consent is always respected.
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- Remarkably, some important aggregation problems that are not strongly sequentially unanimity consistent satisfy the weaker requirement that there exists some decision path along which unanimous consent is always respected.

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An open problem is a general characterization of all weakly sequentially unanimity consistent aggregation problems.
Thank you!